Chap 21 - Cost minimization

Cost minimization

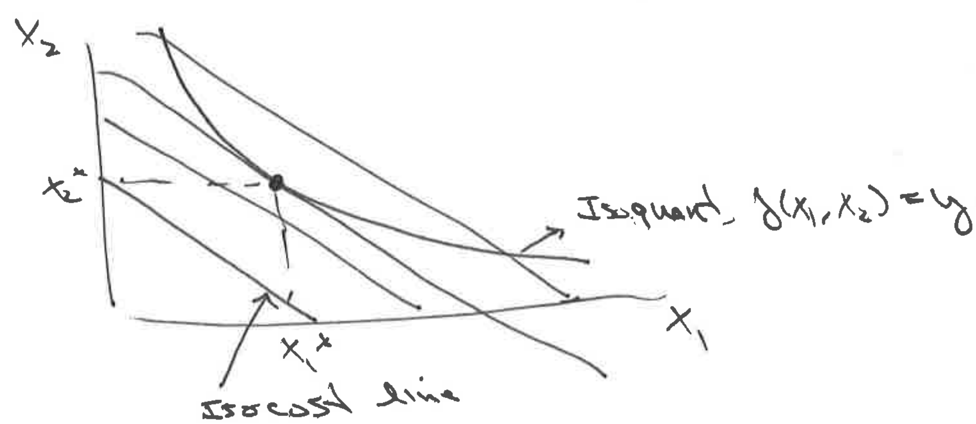
→ The problem

$$\min_{x_1, x_2} \; w_1 x_1 + w_2 x_2$$

$$\text{s.t. } \underbrace{f(x_1, x_2) = y}$$

min cost such that output hits target of y

Solve w/ graphical reasoning:



→ isocost line → line along which diff. combinations of x_1, x_2 give same cost

$$\text{cost} = k = w_1 x_1 + w_2 x_2$$

$$\Rightarrow \text{isocost is } x_2 = \underbrace{\frac{k - w_1 x_1}{w_2}}$$

a straight line w/ slope $= -\frac{w_1}{w_2}$

→ want lowest isocost line subject to touching isoquant

⇒ tangent line

⇒ slope isocost = slope isoquant at optimum

$$\Rightarrow \quad -\frac{w_1}{w_2} = -\frac{MP_1}{MP_2} = TRS$$

$$\Rightarrow \quad \underbrace{\frac{w_1}{w_2} = \frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)}}$$

x_1, x_2 that solve this and $f(x_1, x_2) = y$ are the solution to the cost min problem,

$x_1(w_1, w_2, y)$

$x_2(w_1, w_2, y)$

Solving w/ calculus:

→ use Lagrangian to deal w/ constraint:

$$\mathcal{L} = w_1 x_1 + w_2 x_2 - \lambda\left(f(x_1, x_2) - y\right)$$

remember how you set this up depends on minimization or maximization problem

FOCs:

1) $\dfrac{\partial \mathcal{L}}{\partial x_1}$: $w_1 - \lambda \dfrac{\partial f(x_1, x_2)}{\partial x_1} = 0$

2) $\dfrac{\partial \mathcal{L}}{\partial x_2}$: $w_2 - \lambda \dfrac{\partial f(x_1, x_2)}{\partial x_2} = 0$

3) $\dfrac{\partial \mathcal{L}}{\partial \lambda}$: $f(x_1, x_2) - y = 0$

$$\frac{(1)}{(2)} \Rightarrow \frac{w_1}{w_2} = \frac{\cancel{\lambda} \dfrac{\partial f(x_1, x_2)}{\partial x_1}}{\cancel{\lambda} \dfrac{\partial f(x_1, x_2)}{\partial x_2}}$$

$$\Rightarrow \underbrace{\frac{w_1}{w_2} = \frac{MP_1}{MP_2}}$$

same as graphical sol'n

Example: cost min. problem

let $f(x_1, x_2) = x_1^{1/2} x_2^{1/2}$

$y = 9$, $w_1 = 1 = w_2$

FOC's $\Rightarrow$ $\dfrac{w_1}{w_2} = \dfrac{MP_1}{MP_2}$

$$\frac{1}{1} = \frac{\cancel{\tfrac{1}{2}} x_1^{-1/2} x_2^{1/2}}{\cancel{\tfrac{1}{2}} x_1^{1/2} x_2^{-1/2}}$$

$$1 = \frac{x_2}{x_1}$$

$$x_1 = x_2$$

$\Rightarrow$

$f(x_1, x_2) = 9$

~~$f(x_1, x_2)$~~

$x_1^{1/2} x_2^{1/2} = 9$

$x_1^{1/2} x_1^{1/2} = 9$

$x_1 = 9 = x_2$

Cost minimization in the short run

→ at least one factor fixed

→ problem:

$$\min_{x_1} \; w_1 x_1 + w_2 \bar{x}_2$$
$$\text{s.t. } f(x_1, \bar{x}_2) = y$$

→ sol'n → just find x_1 from

$f(x_1, \bar{x}_2) = y$

$$\Rightarrow x_1 = x_1^s(w_1, w_2, \bar{x}_2)$$

$$x_2 = \bar{x}_2$$

$$\cancel{c_s(y, \bar{x}_2) = w_1 x_1^s(w_1, w_2, \bar{x}_2, y)}$$

$$c_s(y, \bar{x}_2) = w_1 x_1^s(w_1, w_2, \bar{x}_2, y) + w_2 \bar{x}_2$$

→ sometimes just write SR or LR costs as $c(y)$ → b/c take w_1, w_2 as given in competitive environments

Some definitions that are useful

→ Fixed costs – costs paid regardless of output level

→ quasi-fixed costs – costs paid ~~if~~ only if produce output, but they don't depend on amount of output (other than non-zero)

→ sunk costs – unrecoverable fixed costs

- sunk costs are things that shouldn't enter into the optimization problem
  - → i.e. don't fall for the "sunk cost fallacy"